A Computer Science Paradigm in Fundamental Physics
(I) Outline of a Critique

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Abstract

This paper is a synopsis of a study aiming towards a critical assessment of the extent to which computer science concepts and methods can be fruitfully applied to problems in fundamental physics.

Quite a few research avenues are now investigating models based on discrete degrees of freedom underlying the conventional account of microphysical reality in terms of the particles and fields of the Standard Model. These attempts are generally motivated by the problems of quantum gravity, and the hope would be to resurrect a fully unified model of all microscopic physics, including gravity. Of course, no convincing realistic models exist as yet, and are perhaps not likely to appear in the near future. Nevertheless, discrete degrees of freedom, whatever their particular nature, are amenable to computer science methods.

It will be argued that if contentions such as these are taken seriously, or if they turn out to describe the actual state of affairs as regards the constitution of reality, then there might be new conclusions to be drawn as to the nature of fundamental physics. One such conclusion could be the non-existence of any unique characterization of the basic degrees of freedom. In order to reach such conclusions,

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a more detailed correspondence between concepts in physics and concepts in computing science must be developed. Investigating such correspondences will be the subject of subsequent papers in the present study, thus hopefully shedding light on several ontological and epistemological problems in fundamental physics and natural philosophy.

1 Introduction

In physics we are accustomed to regarding matter as discrete, made up of atoms, nuclei, electrons, quarks, et cetera at ever more minute scales. Theoretical research in elementary particle physics and quantum gravity is pointing towards a discrete nature of spacetime itself, or even relegating spacetime to a secondary phenomenon, deriving from more fundamental, possibly discrete, degrees of freedom.

Perhaps not surprisingly, in contemporary theoretical physics it has become fashionable to speculate about the universe being a computer [1, 2, 3].[1] Thoughts of this kind can be traced back at least to a monograph by Konrad Zuse [4]. Of course, such statements should not be taken too literally.[2] What is meant is that there might be some fundamental model of physical reality in which the basic degrees of freedom are discrete and interacting in a way that can be understood as computation. Physical processes should then correspond to computational processes and the laws governing them as being algorithmically defined. Thus, the much sought for "Theory of Everything" (if there is such a theory) might eventually be described as a set of computational rules for an ongoing computation in some, as yet unknown, discrete substratum. Our particular universe would in such a description correspond to the enacting of a particular algorithm, or perhaps more adequately, of a distributed set of parallel algorithms. Physical reality, as we normally perceive it, would emerge in a large scale limit.

A variant on this theme is to entertain ideas of reducing all of physics to a theory of information. This kind of thinking arose in the late 1970's and the 1980's when connections between physics, computing science and information theory was researched. See, for example, several articles in [6], in particular the keynote paper [7] and the collection of papers in [8] and [9].

Yet another of line of research is "discrete physics" where one hopes to solve the quantum gravity problem by working bottom-up, so to speak, from a discrete substructure, hoping to resurrect (quantum) general relativity. These approaches include[3], Penrose spin networks

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[1] A more extensive list of references will be given in due course.

[2] Given the ubiquitousness of computers, it is nothing strange that the computer is used as metaphor for the universe. That has happened before with dominating technologies influencing the world view. Some examples are cited in [5] on page 18.

[3] Only original references or references to reviews will be listed here.

[10] and their subsequent development in conjunction with loop quantum gravity [11], Regge calculus [12], causal sets [13], casual dynamical triangulations [14] to name a few [15]. Although computational notions are not exploited explicitly, they are implicit as soon as discrete degrees of freedom are considered.

However, equating physics with algorithmic computation or a theory of information stretches both conceptual models beyond what is scientifically known. Can all of physics really be modeled algorithmically? Is general computation powerful enough to encompass all physical action? Is "information" really the basic building block? The nature of physical reality and the extension of concepts such as algorithm and computation is up for discussion. Clearly, any attempt to base physics on underlying discrete degrees of freedom and consonant algorithmics, faces a really tall order. Unless continuum quantum field theory in the form of the Standard Model can be derived, not to say independent experimental corroboration secured, it will be hard for such a theory to claim any success.

The number of articles and books in some way relating to this research (and speculation) is enormous. It is scattered over a wide spectrum of journals, conference proceedings and books ranging from physics, over computer science into philosophy. To the extent that this literature can be surveyed at all, it gives a very disparate and unsystematic impression. This is to be expected. It reflects the intriguing nature of the subject, touching on the very nature of reality, and thus being accessible from many compartments of knowledge.

The literature ranges from technical research papers to survey articles and popular articles aimed at a general audience.[4] With all due respect for the efforts of other writers, it is somewhat difficult not to get the impression that the purpose, at least in the general/popular literature, is more often to dumbfound the reader than to inform her. Perhaps this only reflects the enthusiasm of the writer. Not unexpectedly, the spectrum of texts extend from the serious into the wacky. Insubstantial claims are common. Even putting the outright speculative out of sight, it is easy to just dismiss these lines of research as misguided and unlikely to succeed [16].

The purpose of the present work therefore is dual. On the one hand, to critically examine the "universe as a computer" and the "physics as information" circle of ideas, while on the other hand,

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[4] The theme is common in science fiction books and films.

to investigate the extent to which Computer Science[5] methods and concepts *can* indeed be applied to problems in fundamental physics. Computer science thinking is often implicit in the theoretical physics literature and, quite naturally, it surfaces explicitly in research on quantum computation and information theory.

Due to the vast range and scattered nature of the extant literature, it is not feasible to review it on its own premises. Rather, the present study will be organized around a number of themes that crop up time and again in various guises in the literature, and can thus be used as pivots for a critique. These themes are as follows.

(II) Conceptual Roots

(III) Dichotomies of Thought

(IV) Machines and Physics

(V) Consequences for the ToE idea

The numbering here refer to a planned set of papers treating these questions. Below follows a brief outline of the intended contents of these papers.

The second paper

The second paper will try to survey at least parts of the extensive literature with the aim of trying to understand the conceptual roots of the "universe as a computer" idea. The *modern roots*, as it were, can be traced back to the advent of quantum mechanics and to the new theories of computation from the first decades of the 20th century, roughly the period from 1900 (Planck radiation law) to 1936 (Turing machine). Explicit research into the connections between the theory of computation and theoretical physics dates from the late 1970's and early 1980's. See for example [8] and [9]. A full bibliography, which would probably contain several hundreds of papers and books, is not practical, since digging deeper, almost always turns up new references. Original work will be cited to the extent that I am aware of it, and secondary work and reviews will be cited according to a judgment of the relevance to the study at hand.

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[5] What we mean is really Comput*ing* Science, a term used by computer scientist when they want to emphasize that the subject is not physical computing machinery, but rather abstract data and computing processes in full generality. We will, as is customary, continue to use the more common term "computer science" with this understanding.

However, it is also possible to talk about *ancient roots*. As an example, approaching Plato's theory of ideas from a modern point of view hints at an interpretation in terms of the class/object template of Object Oriented Programming. Plato's ideas, or forms, are our abstract classes, and his "shadow" world of reality is constituted of what we would call instantiated objects.

Between the ancient and the modern, we will discuss *10th - 19th century roots*. An example from this period that can be reinterpreted from a modern point of view is Berkeley's idealism [17]. Berkeley's immaterial world of "interacting" spirits can, similarly to Plato's theory of ideas, be thought of in terms of objects interacting via message passing in a virtual reality.

Strange, or contrived, as these examples may appear, one purpose is to direct attention to the remarkable uniformity of human thought once particulars of terminology, and historical circumstances are abstracted from. Of course, such modern reading always run the risk of attaching views to thinkers of earlier times that was unknown or alien to them. However, as sometimes pointed out, the past exists as records in the present, and just as it is interesting (but in principle impossible) to try to reconstruct what an author actually thought, it is just as interesting to let the text stand on its own, reinterpreting it in the light of new knowledge. We know much more about nature than the ancients did, but the questions about form, substance and matter that they struggled with, are still very much with us.

The third paper

The third paper will treat three major dichotomies of human thought

(1) The continuum - discrete dichotomy
(2) The classical - quantum dichotomy
(3) The syntax - semantics dichotomy

In studying the "universe as a computer" idea, clearing away incidental speculation superficial details of exposition, it is clear that the central issue is precisely:

The nature of the fundamental degrees of freedom constituting our physical universe and the dynamics governing them.

The "nature of the fundamental degrees of freedom" is an experimental question in the last analysis. We simply do not know what they are. However our description of them will have to be in terms of language; natural, formal and mathematical, and our understanding will have to be theoretical and semantic.[6]

The language of physics, in particular classical physics, is traditionally continuum mathematics. Quantum physics, quantization of certain physical quantities notwithstanding, also relies on continuum mathematics. On the other hand, the language of computer science is discrete mathematics. Now of course, a real computer is a human built artifact, a machine engineered to work in a discrete fashion. The underlying physics, if it is a mechanical computer is continuum classical physics. If it is an electronic computer, the underlying physics is quantum physics. Thus, we have two interfering dichotomies: the one between the *continuum* and the *discrete*, and the one between *classical physics* and *quantum physics*. Neither of these can be discussed without first make a clear distinction between a *physical system itself* and a *model* of it. Likewise, we must distinguish between a real computer and our, more or less, abstract model of it. During this analysis it will become apparent that a third dichotomy, the one between *syntax* and *semantics* interferes with the other two. Thus, our language-based descriptions and understanding is riddled by three deep dichotomies of thought. Whether these are just dichotomies of thought, or if they run to the core of reality, is an interesting question that we will eventually have to leave open.

Note that the use of the term "dichotomy" is not meant to indicate that eventually we would have to make a choice. Not at all. It is well known that discrete and continuous entities in mathematics are strongly wed. The same goes for the classical objects and quantum objects in physics. The continuous eigenvalue equations that crop up in quantum mechanics yield discrete eigenvalues. In computer science, semantics of programming languages are formulated as syntactical rules. And as already noted above, we must use classical language to describe quantum phenomena.

Dichotomies like these run way back in history. Modern doubts about the fundamental status of the continuum go back at least to the advent of quantum mechanics.[7] It seems that the best argument

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[6] Among physicists, Niels Bohr stressed the role played classical language even in describing quantum phenomena. *"... However far the quantum phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms. ..."* See [18], page 100 in particular for further discussion.

[7] Most notably by A. Einstein as referred to in [19], page 27.

for the continuum is simply one of convenience and pragmatism. Continuum mathematics is just incredibly powerful in its application to natural processes. On the other hand the best argument against the continuum is its sheer incredulity as an inherent property of reality.[8] This was succinctly formulated by R. Feynman in a famous quote [20].

> It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of spacetime is going to do?

Research into black hole physics does indeed throw doubt on the assumption that the number of degrees of freedom in a spatial region is infinite and scales as the volume [21]. This topic will be returned to briefly in section 5 of the present paper.

A key issue is whether nature "computes" with infinite precision real numbers or not. This is a question that seems to be inherently inaccessible to experiment, since any measurement, classical or quantum, always yields a finite precision answer, i.e. a rational number.

The fourth paper

The fourth paper is the central one. An attempt will be made to clarify the concepts of physical systems and their models as actually used in physics. The same will be done for real computing machinery and the abstract models of them as actually used in computer science. In this way differences and similarities can be highlighted. Clearly, the backdrop of the dichotomies investigated in paper III will be central for this undertaking to advance beyond the trivial.

We do this in order to systematically discuss the question as to the algorithmic nature of all physical action, the limits of computation and different version of the Church-Turing thesis. This will require a deeper understanding of the concept of an algorithm. We will briefly return to this topic in section 3 of the present paper.

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[8] However, it must be clear that up till not very long ago, the continuum nature of spacetime right down to the infinitesimal was considered self evident. Indeed, it is probably still the majority view.

The fifth paper: Thesis on discrete computational microphysics

The fifth paper will draw on all the previous and will contain a critique of some of the major ”universe as a computer” schemes that has been put forth. More important though, will be an attempt to draw general conclusion about the ToE idea as such from taking computer science methods and concepts seriously in fundamental physics.

As already noted, there are quite a few ”discrete physics” approaches to the problems of microscopic physics, some of them explicitly computational, some only implicitly computational. Of course, if the idea is completely misguided, all of them will turn out to be wrong. One scenario (the conventional) is that one of them, or one such not yet formulated, will turn out to be correct. However, and this will be argued on the basis of our investigation, if the idea of a discrete computational substructure is sound, then most probably, several of them are likely to be correct. Thus, insofar as the present work has any (hypo)thesis, it would be

If there is a discrete computational substructure,
then it is not unique.

Formulating a thesis might seem to run counter to the stated aim of the present work of providing an overview and a critique. However, viewed as a hypothesis to serve as a guide in the investigation, rather than a thesis to be defended at all costs, I think it is useful.[9]

The present paper

The present paper serves as an introduction and a plan for the whole project. Some points mentioned in the foregoing synopsis will be further elaborated.

In section 2, the stage will be set by considering a hypothetical scenario, the significance of which will be returned to in the very last paper on ToE’s.

In section 3 and 4 respectively, some general comments about computer science and physics will be made that will be elaborated much more fully in papers III and IV.

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[9] This is one point of contact with certain perennial debates in ontology [22], in particular the issue of structural realism.

In section 5, a preliminary formulation and discussion of "A Computer Science Paradigm" will be made. Furthermore, a tentative framework for discussing the possible non-uniqueness of a fundamental description will be set up.

The proposed study is clearly inter-disciplinary in nature, involving as it does theoretical physics, theoretical computer science and mathematics. Concepts and results from high energy physics and computer science that are well known among the experts will be outlined in order to fix the framework for the discussion, and to make the argument accessible to both computer scientists and physicists and other interested readers.

Finally, it must be stressed that the purpose of the present study is not to put forth yet another speculation. Quite to the contrary, the purpose is to provide a critique.

Now, to set the stage, consider the following hypothetical scenario.

2 What if?

Consider a scenario in which the development of computer science had occured in a quite different scientific environment. What if David Hilbert had never posed his "Entscheidungsproblem"? Going back in time, what if he had never proposed his axiomatic formal approach to mathematics. He didn't because the mathematicians of the last part of the nineteen century did not worry about infinitesimals and the foundations of analysis. They went on doing calculus in the time honored Newton-Leibniz-Euler style. What if rigor had been of no concern. If mathematics had developed, not in the "Euclidean" definition-theorem-proof style, but instead in a more calculational, "Archimedean" style? What if a influential hypothetical mathematician had claimed (paraphrasing Paul Dirac) "I'm not interested in proof, just how mathematics works." If Gottlob Frege had not drilled deep into the concept of number? What if Bertrand Russell and Alfred North Whitehead had never attempted founding all of mathematics on logic, and Kurt Gödel had never derived the incompleteness results. Alonzo Church, Alan Turing and the other mathematicians and logicians would have had no reason to study the limitations of computational processes.

What if electronic digital computers and *a* (i.e. just one) high level programming languages had developed in such a historical environment. Certainly you don't need $\epsilon - \delta$ definitions of derivatives in order to investigate and formalize electrodynamics and quantum phenomena. Nor is it needed to manufacture solid state electronic devices.

At some stage the computer engineers might start to ask themselves: What is the origin of *the programming language*? From where does it get its power? Can we understand it on a more fundamental level? An investigation into the foundations of computing starts.

Would it be entirely impossible to imagine that such an investigation would turn up fundamental mathematical models of computation which would soon be proved to be equivalent in power and all related to an intuitive concept of an algorithm, i.e. the analogue of our Church-Turing thesis?

Thus, in such an historical scenario, one could envision a development from one fixed language of computation towards an intuitive concept of computation capable of being formalized in any of a large (denumerably infinite) set of more or less different languages. Uniqueness of description would be lost.

3 Lessons from computing science

A most remarkable lesson to be drawn from computing science is the fact that general purpose, or universal, computation can be decomposed into just a few extremely simple primitive operations. These primitives can, as is well known, be formulated in many ways. Apart from the "classical" models, Turing machines, Church λ-definability and Herbrand-Gödel recursive functions (for original references, see [23]), computer scientists have invented a range of abstract computing machines, as well as hundreds of programming languages [24]. Common to all formulations devised to capture the essentials of computation is the simplicity of the basic operations.[10]

All that is required to achieve universal computation is a set of discrete degrees of freedom and some simple rules of manipulation [25]. These degrees of freedom can be physical (eventually some physical degrees of freedom are needed in a real computer) but they can also be abstract symbols. As soon as some coding of input and output is agreed on, any recursive function can be computed. Almost as a

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[10] Practical computing machinery and programming languages of course have higher level constructs that facilitates effective programming in various contexts.

byproduct, the well known limitations on the power of computation follows, i.e. the recursive functions form a denumerable set, whereas the cardinality of the set all mathematical functions (from N to N say) is non-denumerable infinite.

An important aspect of this is the isomorphism between the natural numbers and strings over any finite alphabet. General computation is really just string processing. This is one source of both the strength and weakness of computing. Any data that can be encoded in a string of symbols, and any process that can be described as a dynamic processing of such strings, is amenable to computer science tools. Whether this is enough to capture all of physics is unknown.

Algorithms and computational models

As is well known, the concept of an algorithm cannot be defined but it can be characterized [24, 26, 27, 28]. This was clearly understood by the pioneers of computational theory in the 1930's. The models of computation then developed were all attempts to capture in precise mathematical terms what is meant by a computation. As it turned out, the different models put forward were all shown to be equivalent in the sense that they all defined the same set of recursive functions.[11]

All the models were meant to be abstract mathematical models of computation. The Church-Turing thesis identifies the computable functions (according to the models) with what can be effectively computed by following algorithms. It is labeled as a "thesis" as it is generally not considered to be provable since it identifies an intuitive, non-formal concept with an exact mathematical definition.[12]

The thesis as originally formulated, covered algorithmic computations performed by humans, but also "machine" computations.[13] With the advent of electronic computers, the thesis in now almost exclusively formulated as concerning the computational power of digital computers. The issue is much clarified by explicitly disambiguating between calculations by *humans*, *machines* and *physical systems* respectively. It is the latter two kinds of computations that are interesting in our context.

Attempts to explicitly model abstractly the capabilities of machine computations are very few. Arguably, the most thorough is Robin

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[11] See several papers in [29].

[12] For an interesting and different point of view, see [30].

[13] This has been debated. References to the literature will be given in paper IV.

Gandy's analysis taking into account parallel machine computation [31].[14] This analysis is classical.

The general computational capabilities of physical systems has not been investigated to any significant degree. This is once again a place where the discussion is riddled by the dichotomies. It could be argued that any well defined classical physical system is an analog computer, computing its own evolution. Obviously this does not lead very far. What we are really interested in is digital computation. Furthermore, as far as we know today, nature is fundamentally quantum mechanical. Thus it ought to be quantum (digital) computation that we should investigate. In quantum computational theory there are universality theorems (for references see [32]). Together with arguments for quantum computers being able to simulate any quantum physical system, it has been argued [33, 34] that a quantum physical Church-Turing principle unambiguously connects the theory of quantum computation to physics, making all physical action computable. A critical discussion of this contention will be one of the objects of paper IV.

Perhaps it should also be pointed out that as we do not know the final form of the laws of physics, the question as to the algorithmic nature of all physical action is in principle impossible to answer as yet.

Characterization of algorithms

An algorithm consists of a set of instructions for carrying out a certain task. In computer science the task is a computation. The concept can, and must, be further elaborated by the following clauses.

- The set of instructions should be precise and unambiguous. The number of instructions should be finite and each instruction should be finite in length.
- A machine or a human can execute it.
- There should be no room for subjective decisions, appeal to human intelligence or creative intervention of the user.
- It should solve some general problem.
- It need not be phrased in any particular language, programming or natural.

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[14] For once, the secondary literature is scant.

Note that termination is not included among the clauses since not all algorithms are meant to terminate.[15]

This characterization of the concept of algorithms is in a sense traditional, even old-fashioned. The modern concept of algorithm is much wider, including as it does *interactive* algorithms [16], *parallel* algorithms, *non-deterministic* algorithms, *probabilistic* algorithms, *distributed* algorithms, *evolving, self-modifying* algorithms. With some slight modifications, the characterization can be adapted to these cases. For instance, allowing an algorithm to temporarily halt and consult the ”outside world”, interactive algorithms can be included.

Even so, in order to investigate the power of algorithms with respect to physics, we need to remove the human-electronic computer idiosyncrasies from the characterization, and adapt it to general machine computation, in particular to parallel computation. Eventually, an adaption to computations by general physical systems is needed.

Data structures and objects

Computing science is not just about algorithms. Just as important are the concepts of *data structures* and *classes/objects.* Even though data structures and objects can be implemented algorithmically, they are so important as organizing structures, that it makes sense to think of them as fundamental concepts in themselves. In particular, the class/object concept is extremely powerful in describing general entities. Most likely, these concepts, or an elaboration of them, should play a role in a computational approach to physics. It seems that often, when computational thinking is applied to fundamental physics, it is in terms of low level concepts and tools like cellular automata and circuit models. That is unnecessarily restricted. It is quite possible that high level concepts as classes/objects will be useful.

The reason for this is that computer science is very much about modeling reality. Many thinkers, throughout history, have come to the conclusion that there are ”things” and ”relations” between them. Things have ”properties” and ”behavior”. From these simple observations, much of philosophy emanates. Computer Science, pragmatically, takes stock of this and offers various related ways to ”encode”

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[15] An example is Operating Systems.

[16] A special type of interactive algorithms could be *oracle-consulting* algorithms. Such an algorithm could in principle compute an un-computable function by consulting an oracle. The action of the oracle would not be computable of course.

this circumstance. One is "Entity-Relationship" modeling in database theory [35], another is the Unified Modeling Language, or in general Abstract Data Types with concomitant functions.

Category theory is a mathematical language that systematically puts equal weight to "things" (data, objects) and "relations" (processes, morphisms). Indeed it puts perhaps even more weight on relations. Category theory also encourages abstraction, and it is therefore an anti-dote to the desire to "peek into" objects. Focus is on the properties and behaviors of objects and their interfaces towards the environment, not on their internal machinery. This kind of thinking is probably under-exploited in theoretical physics [36].

How powerful is computer science?

A most crisp characterization of theoretical computer science is to say that it is

a general science of data and processes.

Any *data*, any *process*. At least as long as they are discrete and classical.

If it were not for these two last caveats, there would be nothing left to discuss, and yes, for all practical purposes, the world would be a computer. Again we are confronted with the above mentioned dichotomies.

A brief discussion is warranted. Classical physics, which is continuous in nature, and the inherently discrete classical computational theory does not go very well together. There is an "impedance mismatch" so to speak, coextensive with the continuous - discrete dichotomy. True, in practice, classical physics can be approximated and simulated on classical computers by way of discretization, but we are interested in questions of principle here. Comparing classical computation with quantum physics, the mismatch seems to be even more pronounced. There is however a good side to this mismatch. Classical algorithmic problems can be solved without any regard to physics. There is a clear distinction between software and hardware, and the existence of generally applicable computational primitives, allows for efficient algorithm construction. Implementation is possible without checking physical realizability at every step.

Upon comparing quantum computation theory to quantum physics, no such mismatch can be seen. Any quantum physical system can in principle be efficiently simulated on a quantum computer, or so present day heuristic understanding claims. Vaguely speaking, quantum physics is thus essentially coextensive with quantum computation. However, now it seems that not having the above mentioned mismatch, nor do we seem to get the "decoupling", as it were, of algorithmics from the physical substrate. In a way, quantum computation is not a theory of computation at all, at least not in the sense of classical computation theory. Although it is perhaps too early to say for sure, the very difficulty in finding non-trivial quantum algorithms might be an effect of this absence of powerful yet simple and generally applicable primitives that are decoupled from the physical implementation.[17]

Needless to say, this discussion is tentative, and will be further pursued in paper IV.

Physical systems, machines and models

It is a common observation that there is a close correspondence between physical systems and computing machinery, or phrased differently, between physical processes and computations.[18] Providing a computer with input that conforms to appropriate requirements, it produces output if the result is computable. We have an *input-transformation-output* model. Tentatively defining a physical system as a piece of the world[19] with a well defined interface towards the environment,[20] it is quite clear that physical systems can be understood in this way too. Supplying a physical system with input through its interface, and letting it evolve, output can eventually be extracted. This argument is admittedly vague, to make it more exact we must make a distinction between the real physical system itself and our model of it. The same goes for human built machines. A machine is a physical system, specifically engineered to function in a particular way. A computer then, is a special kind of machine. One feature of computers that sets them apart from other machines, is their capability to compile a high level description of an algorithm into low level basic operations.

Modeling physical systems computationally can be done in differ-

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[17] There is however an extensive literature on various aspects of "quantum programming".

[18] See for instance [37].

[19] More precisely: a bounded region of space and a finite duration in time.

[20] Subject to quantum considerations.

ent ways, just as different algorithms can generate the same input-output function. Obviously, the executing algorithm is not the same as the evolving physical system. One way of keeping these apart is to disambiguate between the unanalyzed dynamic evolution of the system itself[21] and the several layers of abstraction needed in a computational description: the abstract simulating algorithm and various virtual machines executing it and the physical processes in the simulating hardware itself [38].

We are touching on questions already researched in the context of robotics, cybernetics and AI [39]. On the one hand we have the problem of modeling natural systems computationally, and on the other hand using such models to build artifacts, acting or behaving like natural systems, *robots* as it were. Although clearly related to our subject, and undoubtedly there are lessons to learn from this kind of research, our focus is different. Rather that modeling physical systems computationally, we would like to investigate fundamental computational structures underlying the basic particles and fields making up the space, time and matter of the universe.

4 Theories of fundamental physics

Whereas computing science lives with a large variety of different formulations of the same underlying concepts, fundamental physics is much more restricted in its use of different formulations. The underlying concepts of contemporary fundamental physics is *particles* and *fields* and overarching principles such as *the equivalence principle*, *the relativity principle*, various *gauge principles*, the principles of *quantum mechanics*. All of this is merged into the very succesful schemes of Quantum Field Theory and General Relativity. The unity of theoretical physics is really remarkable, concepts such as energy apply across the field. The mathematical formulation of these principles is remarkably stable. The number fields are $\mathbf{R}$ and $\mathbf{C}$, there are manifolds, groups, algebras and partial differential equations - in short it is analysis, perhaps with a sprinkle of discrete mathematics.

Now we have a problem. Are the above mentioned concepts and principles really the fundamental ones? Or are they just intermediate steps towards a full understanding of the fundamental nature of reality? No one knows of course, but the consensus view among theo-

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[21] That is, the system computing its own evolution.

retical physicists seem to be that, although up to the present time the electron, to pick a concrete example, is a truly elementary particle, it will eventually have to be subsumed into a more general framework of some kind. Present day Standard Model & Big Bang/Inflationary cosmology is merely a step towards something more fundamental. This framework would then be the sought for ”Theory of Everything”, or ToE for short. Of course, there are dissenting views, not everyone believes in the existence of a ToE. The conventional ToE is reductive, where higher level phenomena are explained, at least is principle[22] in terms of lower level constituents and dynamics. A different, perhaps more multi faceted, notion of a ToE is developed in [40].

The problems of high energy physics

The development of physics since Newton is often described as a series of *unifications*. Newton himself unified heaven and earth by showing that the same force (gravity) and same laws of motion (mechanics) governed the motion of the moon around the earth and bodies moving near the surface of the earth. The second big unification was the unification of electricity and magnetism in the course of the 18th and 19th centuries, culminating in the electrodynamics of Maxwell.

The discrete, atomic nature of matter was discovered in the course of the development of the science of chemistry, thermodynamics and the kinetic theory of gases and in the study of electric discharges in vacuum tubes.

The structure of the atom and atomic phenomena like emission and absorption of radiation were understood in terms of the new theory of *Quantum Mechanics*. Nuclear phenomena were understood in terms of the *weak* and *strong* nuclear forces. These forces were unified in the 1970’s with electromagnetism into what is now called the Standard Model of Elementary Particles.[23]

The Standard Model of particles and fields encodes all our experimental knowledge about the ”elementary” particles and the three spin-1 gauge field theory forces acting between them. It is a renormalizable quantum field theory, and as a such, pragmatically combines the special theory of relativity and quantum mechanics. It leaves gravity out, and that is one of the sources driving research in fundamental physics to go beyond it. Another source is the fact that it contains

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[22] Since the actual calculations might be untractable.

[23] A canonical textbook reference is [41].

many parameters, like masses of particles, coupling constants and even the fundamental constants as c and $\hbar$, that are left unexplained. No explanation is given for the gauge group $SU(3) \times SU(2) \times U(1)$. It could be said that the Standard Model just "parameterizes", although with a high degree of understanding, our experimental and observational knowledge in fundamental physics.[24]

The Standard Model leaves gravity out. Therefore, the next step[25] on the unification ladder would be the unification of all forces and particles, including gravity. There are mainly two approaches towards this problem. One is the search for a theory of quantum gravity [11, 42]. The other is the approach through string theory [43, 44].

Both of these approaches combines speculative and conservative ideas. String theory could be said to be conservative in that it generalizes point particle quantum field theory into string field theory. It is speculative in that it claims to be a ToE (or *the ToE*) without any experimental corroboration.

Quantum gravity is conservative in that it builds on attempts to quantize general relativity. It is speculative in that these attempts leads to philosophical questions about the nature of space and time.

If there is a theory of everything, what does it describe? Is it a description of our particular universe, perhaps the only one possible? Or is it a description of a class of universes, of which our particular one is but an instance? Either way, the theory at least describes some more or less concrete structure with dynamical laws.

The absence of substance

Macroscopically, matter is tangible. We can grab hold of a chair, and there seems to be no doubt about its concrete physical substance.

But as we study matter, discovering the atom, made of nuclei and electrons, nuclei made of nucleons, which in their turn are made of quarks, it is clear that the tangible matter disappear. What is a quark or an electron? For the high energy physicist it is field with a set of properties governed by a set of equations. There is no matter, nothing to touch - it is just a piece of mathematical formalism. But nonetheless it is real. We now know that the tangible nature of a chair

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[24] It is generally contended that any low energy theory combining special relativity and quantum mechanics must be a renormalizable quantum field theory.

[25] Unless new phenomena are discovered, for example in the upcoming LHC experiments, that inserts new intermediate steps.

is due to the electromagnetic interactions of atoms in the chair and atoms in our fingers, interactions the effect of which are transmitted by our nerves and somehow made conscious to us.

Therefore, it seems that the tangible nature of macroscopic items are not due to any material content, but rather to relations between physical entities (atoms in this case). Focus is shifted from "things" to "relations" between them.

In this sense, an electron is not made of anything. It is even hard to contemplate what that matter would be, because as soon as we postulate some material content for the electron, the question arises as to how to describe that matter. The only tool we have is language, and the choice is between natural language or formal language, the latter of which could be mathematics. Any way, the nature of the fundamental degrees of freedom eludes us, and we are left with just a formal description of them. There simply is no *substance*.[26]

This absence of substance, or downplay of "things" and corresponding increase in emphasis on "relations" is natural from a theoretical computer science perspective. As already briefly mentioned, it can be captured exactly in terms of category theory. In physics however, it is not clear if it simply is a rather trivial description of the state of affairs, or if it offers new guiding principles for fundamental theory [45].

It is clear that these issues are strongly connected to long standing problems in ontology and epistemology, like the idealism - realism dichotomy, and it would be easy to set out on a tangent here. We will try to avoid that, and instead refocus the discussion on what can be said from a computer science perspective. Interesting references can be found in [46].

Why not simply (Natural) philosophy?

Thus it must be clear by now to the philosophically literate reader that many, perhaps most, of the questions raised in this work are not new. They have been discussed in philosophy throughout history.[27] Indeed, this very property of consistency and recurrence of themes in human thought will be one of the meta-themes of the present work. Working physicists tend to be mildly interested in these debates, partly be-

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[26] This term, dating back to the Greeks, in particular to Aristotle, is notoriously ill-defined, and has undergone many shifts of meaning.

[27] More on this in paper II.

cause the often convoluted, even choleric nature of the discussion, and partly because of the participants obvious removal from center stage of physics research.

Why then bring computer science into the picture? I think it is fair to say that, although many intriguing topics has been discussed, philosophy haven't been very succesful in clarifying the issues, at least not in a way that has satisfied working physicists. Quite a lot of nebulous thinking has been generated. Just to take an example, consider the old realism versus anti-realism or materialism versus idealism dichotomies in ontology and epistemology. I will claim that these questions can be clearly phrased within a computer science paradigm, and the hope is that computer science, being much more pragmatic and systematic can actually clarify many issues using its relentless power. Computer science relies on plain, but structured common sense, high level abstraction and low level detailed code grinding, and in the interplay of these three extremes, it becomes a powerful, no-nonsense, tool of analysis.

5 "A Computer Science Paradigm"

It is time for a tentative characterization of the paradigm. We need something which is precise, yet not unduly restrictive. The following six clauses provide a first tentative characterization of the basic conceptual model of a *computer science paradigm* (CSP) in fundamental physics.

1. The fundamental degrees of freedom (*elements*) are discrete.
2. The elements belong to a finite set of *types*.
3. There are relations (*links*) between elements.
4. The links belong to a finite set of *types*.
5. Entities are *structures* built from a finite number of elements related by a finite number of links.
6. Entities evolve and interact in a way that can be described computationally.

Clearly, it is to early to speak of a full fledged scientific paradigm in the sense of Kuhn [47].[28] There is no consensus of view. There are

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[28] See also: Stanford Encyclopedia of Philosophy on the www.

no experimental corroborations or established procedures. The word framework could be used instead of the word paradigm. However, "paradigm" is more focused on its possible future elaboration, and it gives room for a critique of the somewhat disparate notions that it intends to cover. Furthermore, given the broad basis in human philosophical and scientific thought (to be reviewed mainly in paper II of this series), it is justifiable to use the richer notion of a paradigm instead of the somewhat terse "framework".

It is however important to realize that CSP is *not* a proposal for a ToE.

Discussion of the clauses

Note that discrete degrees of freedom does not in itself preclude the use of continuum mathematics. Also, no choice between classical models or quantum models is implied by the clauses. The issue of classical versus quantum degrees of freedom will be discussed below.

Furthermore, the above clauses should be elaborated by explicitly introduce structures built from substructures, and the corresponding derived relations. This is suggestive of some kind of recursive definition of the notion of structure. Indeed, the clauses 1 - 5, which might look restrictive, can be replaced be simply stating that the fundamental structures can be represented as (recursive) algebraic data types. The details rapidly gets technical and will not be further discussed at this point.[29]

Note that some labeling of elements and links are convenient, even necessary, in the case of the dynamics to be algorithmically definable [26]. This can be thought of as internal, relative coordinates of the structures. As we have defined things here, it may seem as we have an inherent duality between *data*, the structures themselves, and *programs*, the algorithms acting on the data. However in the spirit of OOP, we can move the algorithms into the structures themselves (classes) in the form of *behavior* (functions or methods) and defining *message passing* as the mode of interaction between structures. Doing this explicitly, and the above mentioned recursive definition is best left to a technical paper.[30]

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[29] See next note.

[30] Functional programming languages are particularly suited for this, using the fairly standard concepts of algebraic types, abstract data types (ADT's) and classes. These languages have primitives that can be directly applied to the data modeling at hand,

Discreteness

The discrete substructure to space and time is a quite old speculation that goes back to the very beginnings of quantum theory. The infinities of quantum field theory would go away if there was finite high energy-momentum (corresponding to small distance scales) cut-off. Lacking any natural cut-off, the covariant theory of renormalization was developed instead [48].

During the last decade, research into quantum gravity has made the concept of a discrete substructure much more concrete.[31] Of course, one should always keep in mind that these theories have no experimental corroboration as yet and might well turn out to be wrong or misguided. But it is clear that the idea is gaining support. After all, matter is known to be discrete, so why should not the same be true for space and time. The idea of infinitely divisible space has a certain air of incredulity to it.

In all theories of physics except those involving *General Relativity* in some way, space and time are non-dynamical, forming a background for the dynamical quantities. In general relativity, space and time are themselves dynamical, interacting with other quantities, and thus subject to quantization and discreteness. The problem is that no consistent agreed-on, experimentally verified, theory och quantum spacetime exists as yet. Perhaps the closest touchdown with quantum spacetime is the heuristic theory of quantum black holes [50] which predicts some kind of breakdown of spacetime at the Planck scale.[32]

The resistance towards a fully discrete description of physical reality is due to the extreme effectiveness of continuum mathematics in physics and the problems of regaining continuum physics in appropriate limits. Even though quantum mechanics describes discrete structures, it still uses continuum mathematics. But this might be a consequence of the fact that we are approaching microscopic scales from macroscopic scales. The main approaches to high energy processes and thereby, more minute distance scales, are guided by extrapolating successful theories into ever shorter distances. In order to reach a fundamental theory, a leap into fully discrete mathematics might be

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without the need to write explicit lower level imperative code.

[31] Ref. [49] contains further references.

[32] It seems to be generally agreed among high energy physicist nowadays that it does not make sense to speak of a resolution of spacetime below the scale of the Planck length at roughly 10^{-35} m.

needed, where continuum mathematical concepts are downplayed.

Again, the problem we have at hand is not really discretizing existing continuum theories, but rather the inverse problem of deriving continuum physics from fundamental discrete models. This is much more difficult, and intuitively, not knowing the proper restrictions on the discrete data structures, they will in general not yield smooth continuum limits.

Typing of elements and links

Not to make the framework unduly restrictive, it makes sense to allow the elements to simply come in different types. Otherwise we would really just have one kind of fundamental degree of freedom. These types are just *abstract types*, as it were, which do not have to specified until we start to discuss a particular discrete model of reality. The same goes for the links. For the links, the arity of the corresponding relations can be considered to be part of the typing.

Finiteness

Describing physics by continuum mathematics, in practice in terms of field theory, at once entails an infinite number of degrees of freedom in any finite region of space. As already noted, research into quantum gravity throws serious doubt on this. A naive discrete description would imply that the number of degrees of freedom that can be fit into a region of space would scale as the volume of that region. If each degree of freedom is an n-state quantum system, the number of quantum states would scale as n to the power of the volume, and consequently the entropy would scale as the volume.

Although there is as yet no complete theory of quantum gravity, black hole physics has for a long time served as a theoretical laboratory into the properties of spacetime at the most minute scales. These theories suggest that the entropy of regions of space, instead of scaling as the volume, scales as a certain area related to the boundary of the region. This goes under the name of the holographic principle [51, 52, 21]. It puts a much more severe restriction of the number of discrete degrees of freedom. Of course, nothing is known about the nature of these discrete degrees of freedom. However, such an, a priori, unnatural restriction, might point towards a radical rethinking of the nature of the substructure of reality.

Structure and relations[33]

Physical objects are highly structured. From clusters of galaxies all the way down the subnuclear particles and fields of the Standard Model. We do not know how far in these two directions, large scale/small scale, this layer upon layer of structure extends. In the large scale direction, we do not know if the universe itself is the last step, or if it itself is part of a structure of universes. In the small scale direction, we do not know what is the internal structure of the subnuclear particles and fields.

The hope, so to speak, seems to be that eventually we will hit upon a simple, mathematical structure. The experience so far does not really support such a hope. Deeper underlying layers has turned out to be quite complicated physical systems in themselves.

There are two aspects of this that need to be discussed. This will be done in paper IV, so this is just to note them for further discussion.

The first is the importance of dynamics. All working models has dynamical equations of the motion at the core. Many schemes for discrete substructure look very static and non-dynamical. It is as if the algorithmic part of the scheme was lacking.

The second aspect has to do with the level at which we choose to model physical systems. Thinking of a physical system (or rather the model of it) as a part of the world with a well defined interface towards the environment, we can often choose this interface at several different levels of detail, and we still have a working model of the system. Underlying levels of structure are organized as if to provide more comprehensive interfaces at higher levels. This is indeed reminiscent on how higher level primitives in programming languages are constructed in terms of lower level primitives. It must be noted that here we are touching on one of the toughest problems in science, the emergence of higher level degrees of freedom and dynamics from an underlying lower level. This emergence - reduction duality is far from understood.

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[33] In mathematics, these terms are well defined, in philosophy not so, though the meanings are related. I do not want to enter too deeply into these issues presently, but rather strike a balance and use the terms in a pragmatic computer science way. Some references that might be interesting in the context of this paragraph are [46, 53, 54].

Algorithmic behavior and evolution

Having discrete structures, algorithmic dynamics and evolution is natural. It is almost inevitable to think about substructures whose evolution are governed algorithmically and whose interaction is in terms of some sort of dedicated message passing or broadcasting. Dual to the ability to send messages and broadcast, is the ability to receive incoming messages and perceive the environment. These notions are somewhat vague, but the intuition behind them is clear enough (and commonplace in CS) and can be made exact in any definitive data model.

Classical or quantum degrees of freedom?

Contemplating classical degrees of freedom seems to be a non-starter. However, there are also problems with assuming fundamental discrete of degrees to be quantum mechanical, because no matter how much, or how little, of conventional quantum mechanical formalism and dynamics we import, no rationale is given for this. In a way, it is more natural to assume classical data structures. At least then the full power of computer science can be applied.

Clearly, working with classical degrees of freedom at this level amounts to a hidden variable theory. It is known that any hidden variable theory underlying quantum mechanics must be non-local [55]. From the point of view of degrees of freedom modeled as discrete data structures, this is in a way quite natural. An abstract data type (like a graph) has no other inherent measure of ”nearness” than the minimum number of links separating any two nodes. Two nodes that initially are far from each other in this sense, can be made close by adding a new link. Concrete graphs that are embedded in some metric space can of course inherit a measure of nearness from that space, but taking the discrete substructure seriously, we would rather have it the other way around, deriving a metric notion of locality from the abstract data structure. This is of course a formidable problem[34], as along the way, any toy model must derive conventional quantum mechanics[35], and any realistic model must derive (quantum) gravity as well as the quantum field theories of the Standard Model.

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[34] No attempt to claim the universe to be a classical computer can circumvent solving this problem.

[35] An interesting attempt in this direction is [56].

The challenge for any program of this sort, deriving microphysics incorporating quantum mechanics/gravity, is no less than finding an appropriate set of basic discrete data structures *and* the correct algorithms!

Preliminary characterization of worlds

This paper is not meant to be speculative, but it deals with the speculative idea of describing the universe as computational process while taking computer science seriously. A cornerstone of computer science is the correspondence between the intuitive idea of an algorithm and the exact definition of particular computational models. Now for the sake of discussion we will attempt to set up a similar correspondence between an intuitive notion of a world an exact computational model of a world.

How would an informal, but precise, description of a world look? Not a particular universe with its elaborate dynamical laws, but the very concept of a universe - the analogy to the concept of an algorithm. Thus a correspondence like

$$\text{A particular algorithm} \quad \longleftrightarrow \quad \text{A particular universe}$$

is not feasible, since our knowledge and understanding of our universe is still too incomplete for such a correspondence to advance beyond the toy model level. One the other hand, while being just as speculative, a correspondence like

$$\text{The concept of an algorithm} \quad \longleftrightarrow \quad \text{The concept of a universe}$$

can at least be discussed in general terms relying on the knowledge we already have. However, there is an obvious problem. A world is a dynamic object, undergoing change and evolution. It cannot be put in correspondence with an algorithm, which is static, but rather with an algorithm in enaction.

Thus, a *world* can be thought of as a structure capable of undergoing *change*. Viewed in its entirety, this change constitutes the world *history*. The following clauses

1. The structure should be precise and unambiguous and it should have a finite description.

2. Change is governed by rules that are finite in number and finite in length.
3. It need not be phrased in any particular language (mathematical, programming or natural) but any sufficiently precise language containing the appropriate abstractions should suffice.

Corresponding to the intuitive notion of a world, we would have the exact computational concepts

1'. A world is modeled by a set of well defined abstract data types.

2'. Change is governed by a set of programs defined in terms of these data types.

In the case of algorithms we are used to view them from the outside. In the case of worlds, we would rather like to be inside one of them. But we have no problem to imagine ourselves outside the universe while contemplating it. No doubt there are problems with self reference and potential paradoxes here. It might be that when these intuitive concepts are formalized that inconsistencies arise making it impossible to be outside the universe. Note also, that this question touches on one of the long standing discussions within quantum mechanics about the *observer-observed system* duality and the necessity of having an outside observer in quantum mechanics, a notion that is difficult to maintain when what is observed is the universe itself. This discussion will not be pursued here, we just add the following clauses.

4. A world may contain substructures that can be well separated from the rest of the world (the *environment*).
5. Substructures can communicate among themselves and with the environment.

Note, however, that we are not presupposing any form of consciousness for substructures.

The problem of time

From the above discussion it is clear that the issue of time cannot be circumvented. Even though this concept of a world involves "change" and "history" it is far from clear how this relates to "time" in the ordinary sense of the word. An algorithm that is not enacted has no concept of time attached to it,[36] and when it is enacted, time steps are introduced from the outside. If the substructure of reality is described in terms of data structures and the rules of change in terms of algorithms, it is hard to escape the need to enact the algorithms in order for the evolution to unfold.

History could be viewed as a record of all the instantaneous states that the world has passed through in its evolution. If the *history itself* is considered to be *the universe* then of course the problem of time disappears, time being just an illusion.[37] However, replacing the evolving universe by, what a computer scientist would call a *look-up table*, is not very enlightening. Especially not since it seems that we would have to browse through the table in order to experience change. Finally, the problem remains as to how the look-up table was compiled in the first place.

The nature of time is of course one of the most contentious in all of science and philosophy. Not very much will be added to this debate. We seem not to have any similar conceptual problems with space. Time is intrinsically connected to dynamics. From a computer science point of view *space is just data, time is process.*

6 Critique

It is remarkable how far into the intricacies of reality the human mind can see, even unaided by direct experimental input. This is the power of reason. It seems that the world is structured in a way that makes it possible for the kind of rational thinking that we are capable of to unravel its internal workings [58, 59]. Another property of our capability of thinking is that there seems to be no limit as to what we can imagine. These circumstances notwithstanding, observation and experiment is the final arbiter for any theory of the structure of reality.

Thus lines of thoughts are laid down for a critique, and this will be the topics for the following papers.

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[36] Though it might be formulated in a language suggestive of action and change, using words like ”do”, ”while” et cetera. Also, enumeration of expressions in the code evokes the notion of time steps.

[37] For a thorough discussion of such a view on the nature of time, see [57].

References

[1] J.A. Wheeler, In *International Journal of Theoretical Physics* [8], p. 557.

[2] S. Wolfram, A New Kind of Sience (Wolfram Media, 2002).

[3] S. Lloyd, Phys. Rev. Lett. 88 (2002) 237901, arXiv:quant-ph/0110141.

[4] K. Zuse, MIT Technical Translation AZT-70-164-GEMIT (1970).

[5] P.C.W. Davies, Science and the Ultimate Reality. Quantum Theory, Cosmology and Complexity, edited by J.D. Barrow, P.C.W. Davies and J. C. L. Harper, Cambridge University Press, 2004.

[6] W. Zurek, editor, Complexity, Entropy and the Physics of Information, Santa Fe Institute Studies in the Sciences of Complexity, Perseus Books, 1990.

[7] J.A. Wheeler, In Zurek [6].

[8] International Journal of Theoretical Physics, Vol. 21, 1982.

[9] A.J.G. Hey, editor, Feynman and Computation. Exploring the Limits of Computers (Perseus Books Group, 2002).

[10] R. Penrose, Quantum Theory and Beyond, edited by T. Bastin, Cambridge University Press, 1971.

[11] C. Rovelli, Quantum Gravity (Cambridge University Press, 2004).

[12] T. Regge, Nouvo Cimento A 19 (1961) 558.

[13] R.D. Sorkin, Lectures On Quantum Gravity, edited by A. Gomberoff and D. Marolf, Series of the Centro De Estudios Cientificos, Springer-Verlag, 2005, arXiv:gr-qc/0309009.

[14] R. Loll, Lecture Notes in Physics 631 (2003) 137, arXiv:hep-th/0212340.

[15] R. Loll, Discrete approaches to quantum gravity in four dimensions, http://www.livingreviews.org/lrr-1998-13, 1998.

[16] S. Weinberg, The New York Times Review of Books 49 (2002).

[17] G. Berkeley, Treatise Concerning the Principles of Human Knowledge (Various editions, 1710).

[18] M. Jammer, The Philosophy of Quantum Mechanics (John Wiley Sons, 1974).

[19] J. Stachel, In Ashtekar and Stachel [42], p. 13.

[20] R.P. Feynman, The Character of Physical Law (Penguin Press Science, 1992).

[21] R. Bousso, Rev.Mod.Phys. 74 (2002) 825, arXiv:hep-th/0203101.

[22] T.Y. Cao, In Symons [46], p. 3.

[23] M. Davis, editor, The Undecidable, Dover Publications, 2004.

[24] A. Blass and Y. Gurevich, Bulletin of the EATCS 81 (2003) 195.

[25] M.D. Davis, R. Sigal and E.J. Weyuker, Computability, Complexity and Languages (Morgan Kaufmann, 1994).

[26] V. Uspensky and A. Selenkov, Algorithms: Main Ideas and Applications, Mathematics and its Applications Vol. 251 (Kluwer Academic Publishers, 1993).

[27] Y.N. Moschovakis, Mathematics Unlimited - 2001 and Beyond, edited by B. Engquist and W. Schmid, p. 919, Springer, 2001.

[28] A.N. Kolmogorov and V.A. Uspensky, AMS Translations 29 (1963) 217.

[29] R. Herken, editor, The Universal Turing Machine, A Half-Century Survey, Springer-Verlag, 1994.

[30] W. Sieg, Reflections on the Foundations of Mathematics (Essays in Honor of Solomon Feferman), edited by W. Sieg, R. Sommer and C. Talcott, , Lectures Notes in Logic, Association of Symbolic Logic Vol. 15, p. 387, 2001.

[31] R. Gandy, The Kleene Symposium, edited by J. Barwise, H.J. Keisler and K. Kunen, North-Holland, 1980.

[32] M.A. Nielsen and I.L. Chuang, Quantum Computation and Quantum Information (Cambridge University Press, 2000).

[33] D. Deutsch, Proc. Roy. Soc. London A 400 (1985) 97.

[34] D. Deutsch, A. Eckert and R. Lupacchini, The Bulletin of Symbolic Logic 3 (2000) 265, arXiv:math.HO/9911150.

[35] P.P. Chen, ACM Transactions on database systems 1 (1976) 9.

[36] A.K.H. Bengtsson, J. Math. Phys 46 (2005) 042312, arXiv:hep-th/0403267.

[37] S. Wolfram, Phys. Rev. Lett. 54 (1985) 735.

[38] C. Fields, Machines and Thought: The Legacy of Alan Turing, edited by P.J.R. Millican and A. Clarke, p. 165, Clarendon Press, 1996.

[39] S. Norvig and P. Russel, Aritficial Intelligence: A Modern Approach (Prentice Hall, 1995).

[40] D. Deutsch, The Fabric of Reality: The Science of Parallel Universes-And Its Implications (Penguin Books, 1997).

[41] S. Weinberg, The Quantum Theory of Fields Vol. 1 and 2 (Cambridge University Press, 1993 and 1996).

[42] A. Ashtekar and J. Stachel, editors, Conceptual Problems of Quantum Gravity, Birkhäuser, 1991.

[43] M.B. Green, J.H. Schwarz and E. Witten, Superstring Theory Vol. 1 and 2 (Cambridge University Press, 1987).

[44] J. Polchinski, String Theory Vol. 1 and 2 (Cambridge University Press, 2005).

[45] J. Baez, In Rickles et al. [60], arXiv:quant-ph/0404040.

[46] J. Symons, editor, Symposium on Structural Realism and Quantum Field Theory, Springer, 2003.

[47] T.S. Kuhn, The Structure of Scientific Revolutions (The University of Chicago Press, 1970).

[48] J. Schwinger, editor, Selected Papers on Quantum Electrodynamics, Dover, 1958.

[49] L. Smolin, (2003), arXiv:hep-th/0507235.

[50] R.M. Wald, Class. Quant. Grav. 16 (1999) A177, arXiv:gr-qc/9901033.

[51] G. 'tHooft, (1993), arXiv:gr-qc/9310026.

[52] L. Susskind, J. Math. Phys 36 (1995) 6377, arXiv:hep-th/9409089.

[53] J. Stachel, Hilary Putnam. Contemporary Philosophy in Focus, edited by Y. Ben-Menahem, Cambridge University Press, 2004.

[54] J. Stachel, In Rickles et al. [60], arXiv:gr-qc/0507078.

[55] J.S. Bell, Physics 1 (1964) 195.

[56] F. Markopoulou and L. Smolin, Phys. Rev. D 70 (2004) 124029, arXiv:gr-qc/0311059.

[57] J. Barbour, The End of Time (Oxford University Press, 1999).

[58] . P.C.W.Davies, In Zurek [6].

[59] E. Wigner, Comm. Pure Appl. Math 13 (1960) 1.

[60] D. Rickles, S. French and J. Saatsi, editors, Structural Foundations of Quantum Gravity, Oxford University Press, 2006.